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Calculation of Lightning Protection Based on an Electrostatic Model of Leader Attraction

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Abstract—A new method of lightning protection has been developed in which the lightning leader is attracted to ground objects as a result of the electrostatic interaction between the leader charge and induced charges on ground objects.

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Lightning breakthrough via protection systems brings vast damage to industrial objects despite formally strict correspondence of the protection project parameters to existing technical rules. The discrepancy is related to the fact that experimental investigations of the physics of lightning, which are mostly restricted to the final stage of leader propagation, only allow the zones of protection to be determined while not taking into account the phenomena of lightning attraction and selective damage. The development of adequate calculation methods is hindered by a complex, nondeterministic character of lightning leader propagation, while most promising methods of simulation modeling are far from being implemented in practical engineering calculations.

Specific features of the task of lightning protection allow the modeling of leader propagation to be simplified, since the characteristics of statistical (average) lightning are of interest while random factors can be ignored. Experimental investigations include hundreds of runs for each particular position of a high-voltage electrode. As a result, the average leader propagates along the electric field lines so that, despite the whole variety of trajectories, the average direction is vertical [1].

Grounded objects attract the lightning. Observations show that the attractive area is determined by the dimensions of an object (primarily, by its height). Physical explanations of the mechanism of attraction are based on the electrostatic interaction between the leader charge and induced on-ground object charges [1, 2].

According to a hypothesis that was recently formulated by Aleksandrov [2], the development of a lightning leader in the direction of each line of an electrostatic field is equiprobable. In this case, the probability of lightning breakthrough to a rod-protected object can be expressed as [2]

$$p_a = -\frac{Q}{q}, \tag{1}$$

where q is the leader charge and Q is the charge induced in the object with protective rods. It is of interest to consider the surface of $p_a = \text{const}$ (lightning capture zone). The minimum distance from this surface to the rod determines the streamer zone. Since the process of leader attraction proceeds until attaining the capture zone, its area provides a lower estimate of the area of lightning attraction. Then, the final jump stage of leader development begins, which is controlled by the laws of gas discharge. The breakdown voltage of all insulation gaps is assumed to be constant. The lightning strikes an object under consideration if the minimum distance from some point of the capture zone to the object is shorter than that to the rods and the point occurs in the breakthrough zone. The ratio of the breakthrough zone area to the total capture zone area determines probability p_b, so that the total lightning breakthrough probability is $p = p_a p_b$.

Unfortunately, the validity of relationship (1) was not proved in [2] and, hence, remained a hypothesis. The present work was aimed at developing and verifying the effectiveness of a method of lightning protection system calculation based on Eq. (1).

In order to implement the given method, it is necessary to solve two problems of mathematical physics, The first problem consists in determining induced charge **Q** and reduces to calculation of the electrostatic field of the leader for a given three-dimensional (3D) object and a lightning protection system of arbitrary configuration. Since these calculations have to be carried out several thousand times for various positions of the leader, standard computational techniques are ineffective or not at all applicable. The second problem is to construct the surface of equal induced

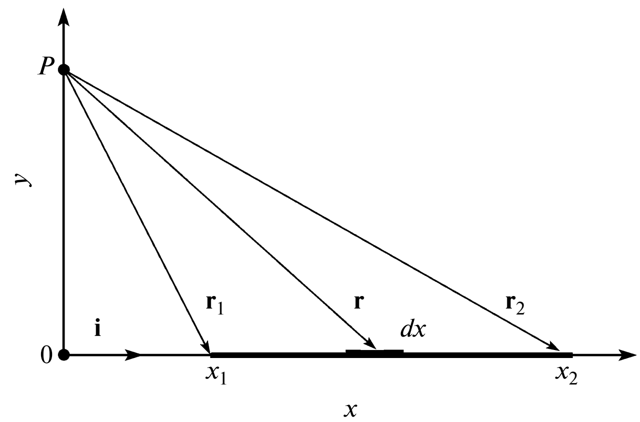

Fig. 1. Scheme of determining the potential coefficient of a rod.

charge (or partial capacitance between the leader and object). This task has not been posed previously and can be classified as an inverse problem of mathematical physics.

We propose the following approach to multiply repeated calculations of an electrostatic field of a complex system of electrodes for a variable position of the field source. Let leader charge q be concentrated in the lightning head. This charge is related to vector $\mathbf{Q}$ of induced charges on elements of the object by the following matrix equation:

$$\mathbf{AQ} + \mathbf{B}q = 0,$$

where $\mathbf{A}$ is the square matrix of mutual potential coefficients between elements with rods, $\mathbf{B}$ is the column vector of mutual potential coefficients between the leader and elements of the object, and it is assumed that the object and rods are either grounded or their potential (working voltage) is negligibly small compared to that of the leader.

The vector of induced charges is expressed as

$$\mathbf{Q} = -\mathbf{A}^{-1}\mathbf{B}q.$$

Substituting this expression into Eq. (1) yields

$$p = \frac{-\sum_{i=1}^{N} Q_i}{q} = \sum \mathbf{A}^{-1}\mathbf{B} = \sum_{i=1}^{N}\left(\sum_{j=1}^{N} A_{j,i}^{-1}\right)B_i = \sum_{i=1}^{N} C_i B_i = \mathbf{CB}, \quad (2)$$

where $\mathbf{C} = (C_1, C_2, ..., C_N)$ is the row vector with the ith element representing a sum of the ith column of matrix $\mathbf{A}^{-1}$ and N is the total number of elements in the object with rods. Vector $\mathbf{C}$ has to be calculated only once, which increases the efficiency of the multiply repeated computational procedure for variable position of the leader.

Let us determine the potential coefficients constituting matrix $\mathbf{A}$. For this purpose, it is convenient to subdivide the object into rod elements of equivalent radius. Assuming that the charge of each element is concentrated on its axis, the linear charge density can be expressed as $q/l =$ const. For point P at the middle of the mth element, the potential related to charge of the kth rod (Fig. 1) is

$$\varphi = \frac{q}{4\pi\varepsilon l}\int_{x_1}^{x_2}\frac{dx}{r} = \frac{q}{4\pi\varepsilon l}\left|\ln\frac{x_2 + \sqrt{x_2^2 + y^2}}{x_1 + \sqrt{x_1^2 + y^2}}\right| = \frac{q}{4\pi\varepsilon l}\left|\ln\frac{\mathbf{r}_2\mathbf{i} + r_2}{\mathbf{r}_1\mathbf{i} + r_1}\right| = \alpha q, \quad (3)$$

where α is the mutual potential coefficient and $\mathbf{i}$ is the unit vector. The intrinsic potential coefficient of a rod with radius R is

$$\alpha = \frac{1}{2\pi\varepsilon l}\ln\frac{l}{R}, \quad l \gg R.$$

Now let us find the potential coefficients constituting matrix $\mathbf{B}$. For the point model of a leader with charge q concentrated in the head, these coefficients are evident. Indeed, the potential of a point charge is

$$\varphi = \frac{q}{4\pi\varepsilon r} = \beta q,$$

where β is the required potential coefficient. As will be shown below, more complicated models of the leader can be reduced to the point model of a charge concentrated in the leader head, which is necessary for the application of Eq. (2).

Consider a rod-shaped leader of length l and charge q. The charge density is known to increase along the leader [1, 2] and we assume that it varies linearly as $\tau(x) = \tau(l)t$, where $t = x/l$ and $\tau(l) = q/2l$. Let us subdivide the leader into n elements and assume that the charge density within the kth element has a constant value of $\tau_k = \tau_n t_k$, where $\tau_n = q/2l$ and $t_k \in [0, 1]$. The potential created by the leader at an arbitrary point of the system is then

$$\varphi = \sum_{k=1}^{n}\tau_k\alpha_k = \tau_n\sum_{k=1}^{n} t_k\alpha_k = \frac{q}{2l}\sum_{k=1}^{n} t_k\alpha_k = \beta q,$$

where α_k is given by formula (3). Thus, the expression for the potential of a charged rod has the same form as that for a point charge (with a different potential coefficient β), which allows Eq. (2) to be applied.

We then proceed to constructing the surface of equal induced charge (or equal partial capacitance) for an electrode of arbitrary shape in the field of a point source. Since the usual methods of interpolation on a grid are highly laborious and ineffective, we propose to construct the electrostatic field pattern by methods based on solving the Cauchy problem [3].

Let us seek a framework of the surface of an equal induced charge on a set of vertical planes (cross sections). 2D local coordinate system XOY on each plane